Mathematics: analysis and approaches
Standard Level
Paper 2

Name

Date: _____

1 hour 30 minutes

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

exam: 9 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Let $f(x)=\frac{x+8}{x}$ and $g(x)=1-x^2$.

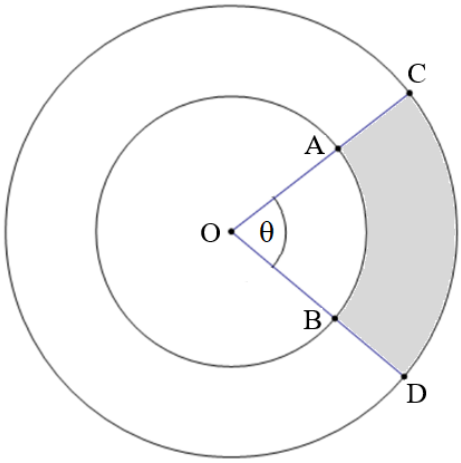
(a) Show that $f^{-1}(x)=\frac{8}{x-1}$. [3]

(b) (i) Write down $\left(f^{-1}\circ g\right)(x)$.

(ii) Solve the equation $\left(f^{-1}\circ g\right)(x)=x$. [3]

2. [Maximum mark: 6]

The diagram below shows two circles which have the same centre O. The smaller circle has a radius of 12 cm and the larger circle has a radius of 20 cm. The two arcs AB and CD have the same central angle θ, where $\theta = 1.3$ radians.



Find the area of the shaded region.

3. [Maximum mark: 5]

Find the coefficient of the x^3 term in the expansion of $\left(\frac{2}{3}x+3\right)^8$.

4. [Maximum mark: 6]

A car begins moving from a fixed point A. Its velocity, v ms^{-1}, after t seconds is given by $v = 8 - t^2 - 8e^{-t}$.

(a) Find the car's displacement from A when $t = 4$. [3]

(b) Find the total distance that the car has travelled from A when $t = 4$. [3]

5. [Maximum mark: 6]

The table below shows the marks earned on a quiz by a group of students.

Mark	1	2	3	4	5
Number of students	8	7	c	9	1

The median is 3 and the mode is 4 for the set of marks. Find the **three** possible values of c.

6. [Maximum mark: 8]

Let $f(x) = x\ln\left(\frac{\mathrm{e}}{2x}\right)$. Point A is on the curve of f where $x = 1$. Point B is also on the curve of f. The tangent to the curve of f at A is perpendicular to the tangent at B. Find the coordinates of B.

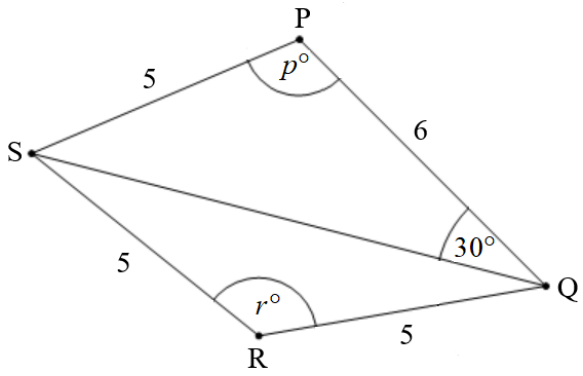
Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

7. [Maximum mark: 15]

The diagram below shows the quadrilateral PQRS. Angle QPS and angle QRS are obtuse.



$\mathrm{PQ} = 6\,\text{cm},\ \mathrm{QR} = 5\,\text{cm},\ \mathrm{RS} = 5\,\text{cm},\ \mathrm{PS} = 5\,\text{cm},\ \mathrm{P\hat{Q}S} = 30°,\ \mathrm{Q\hat{P}S} = p°,\ \mathrm{Q\hat{R}S} = r°$

(a) Use the sine rule to show that $\mathrm{QS} = 10\sin p$. [1]

(b) Use the cosine rule in triangle PQS to find another expression for QS. [3]

(c) (i) Hence, find p, giving your answer to two decimal places.

(ii) Find QS. [6]

(d) (i) Find r.

(ii) Hence, or otherwise, find the area of triangle QRS. [5]

Do **not** write solutions on this page.

8. [Maximum mark: 14]

A commercial plantation grows pineapples that are classified as small, medium or large. The masses of the pineapples harvested in the year 2021 were normally distributed with a mean of 900 grams.

A pineapple is small if its mass is less than 750 grams. For the year 2021, sixteen percent of the pineapples are classified as small.

(a) Find the standard deviation of the masses of the pineapples. [4]

The following table shows the percentages of small, medium and large pineapples grown on the plantation in 2021.

small	medium	large
16%	63%	21%

(b) Given that a pineapple is classified as medium if its mass is greater than 750 grams and less than m grams, find the value of m. [2]

The plantation sends a shipment containing all of its medium and large pineapples harvested in 2021 to a food distribution company.

(c) A pineapple is randomly selected from this shipment. Find the probability that the pineapple is classified as medium. [3]

(d) The food distribution company sells all the pineapples in the shipment. It sells each of the medium pineapples for \$3.30 and each of the large pineapples for \$4.10. The food distribution company paid \$900 for the shipment and makes a profit of \$500 after selling all the pineapples in the shipment. Find the total number of pineapples in the shipment. [5]

9. [Maximum mark: 14]

(a) Given that $h(x)=\dfrac{ax-1}{bx-b}$, find the equation of the vertical asymptote and the equation for the horizontal asymptote for the graph of h. [2]

The vertical and horizontal asymptotes for the graph of h intersect at the point F.

(b) Write down the coordinates of F. [2]

(c) The point $\mathrm{G}(x, y)$ lies on the graph of h. Show that $\mathrm{FG}=\sqrt{(x-1)^2+\left(\dfrac{a-1}{bx-b}\right)^2}$. [4]

(d) Hence, find the coordinates of the points on the graph of $y=\dfrac{4x-1}{2x-2}$ that are closest to the point $(1, 2)$. [6]